Casting Physics Simplified – Part Two

Part one of this paper discussed physics that applies to linear motion, i.e., motion in a straight line. This section of the paper will expand these concepts to angular or rotational movement.

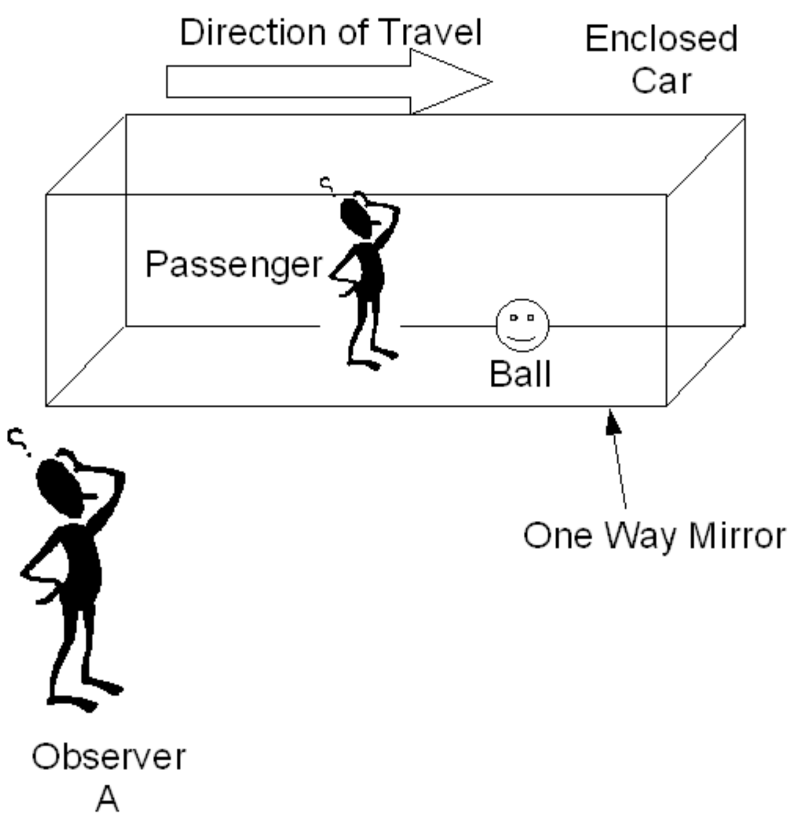
In order to do this I will begin with the concept of a "frame of reference" which will, hopefully, simply our understanding of angular physics.

In addition to angular motion I would like to discuss levers and how they factor into our discussion of fly casting.

Frames of Reference

When we talk about a Frame of Reference we are referring to the position where we chose to watch an activity from. The selection of this position can simplify our understanding of what is happening.

Let's take a relatively simple example. Consider the following figure:



We have an enclosed train car moving at a constant speed of 20 mph. Inside of the car there is a single passenger and a ball. The passenger is unable to see anything outside of the train car and the track it is riding on is perfectly smooth and straight. Unknown to the passenger one of the walls is actually a one way mirror so that our outside observer (observer A), who is standing near the tracks, is able to see everything happening inside the car.

Now suppose that the passenger picks up the ball and tosses it straight up. The ball rises to a certain height and then falls back into his hand. From the passenger's frame of reference the ball appears to travel straight up and straight down. But from Observer A's position the ball appears to travel in an arc as the passenger moves under the ball from the starting position to the end position.

We have defined two frames of reference in this case - that of the passenger and that of observer A. Now let's ask a simple question of both observers - “What is the acceleration of the ball due to gravity as it falls?”

This is an easy determination for the passenger. He/she simply measures the time it takes the ball to fall from its peak to their hand and applies the formula distance = ½ a t^2 to determine a value for acceleration. This is relatively simple because the ball appears to move in a straight line from their frame of reference.

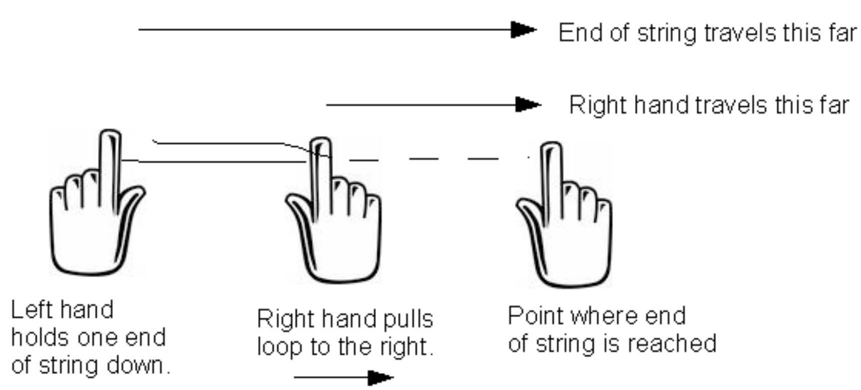
But what is Observer A's perception? From their frame of reference the ball appears to travel in an arc rather than a straight line. Measuring the distance the ball moves from its peak to the point it returns to the passenger's hand can be somewhat difficult and if we simply apply the raw formula that the passenger applied we see that the ball moved much farther in the same amount of time so the value of acceleration must be different. Of course, if we know that the train is moving at a constant velocity and we remember that velocity is a vector quantity we can just subtract the forward portion of movement and then consider just the distance the ball fell and we will get exactly the same answer as our passenger. What would happen if Observer A was also moving? What if his path was different from that of the car?

This is a fairly simple example but we can see that by selecting one frame of reference over another we can make our understanding of a system simpler or much more complex.

Loop Speed

One example of how our choice of frame of reference can simplify our understanding of fly casting is when we look at loop speed.

When we compare the speed of the fly leg to the speed of the loop we see that the fly leg travels at twice the speed of the loop when we are not shooting line. This is explained by the following figure:



In the figure we take a length of string, double it over to form a loop, and lay the loop on a table. With our left hand we pin down one end of the string. We insert our right index finger into the loop. Now we slide our right hand across the table until the string is straightened out. It isn't hard to see that the end of the string traveled twice as far as our right hand so the end of the string must have traveled twice as fast as our right hand. So, if we don't shoot line during our cast, the fly leg travels at twice the speed of our loop face.

But what if we shoot line during our cast? This can be simulated by sliding our left hand to the right but at a slower speed than our right hand. How do we determine the speed that the loop is traveling compared to the fly leg? For purposes of illustration let's say the fly leg is traveling at 2 feet per second and our rod leg is traveling at 1 foot per second.

If we choose a frame of reference in which both hands appear to be moving we would have to measure the total movement of the end of the string and calculate the speed at which it traveled. We would then have to do the same for the loop and then compare the two values. Most of us already know that the speed of the loop is simply the average of the speed of the fly and rod legs. In this case the loop would be traveling at 1.5 feet per second.

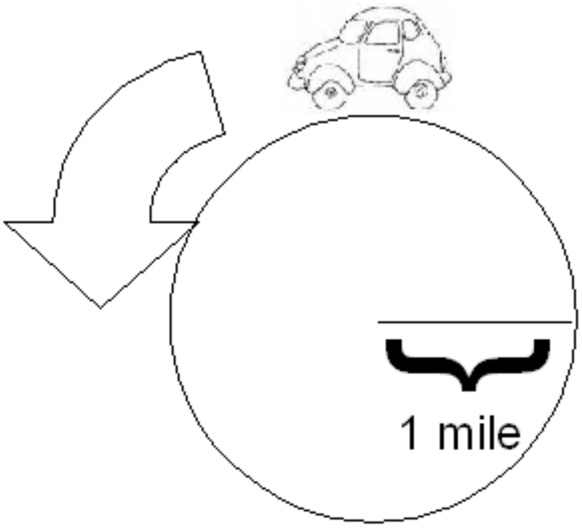
Now if we set up an observation point, or frame of reference, above our left hand and then moved this point in synchronization with the left hand it would appear to the observer that our left hand isn't actually moving. To this observer the fly leg would appear to be moving at 1 foot per second and the loop would appear to be traveling at 0.5 feet per second. Now in order to determine the actual speed of the loop we add the speed of our left hand to arrive at a loop speed of 1.5 feet per second.

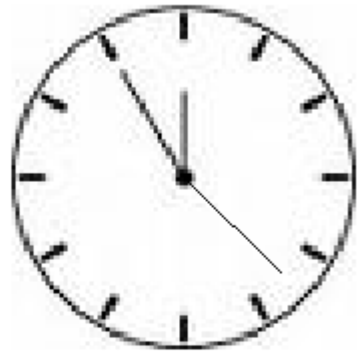
Angular Motion

When we try to analyze angular motions in terms of linear physics we quickly find that things get quite complicated. I mentioned in part 1 that any time an object in motion changes its direction it is accelerating. But if the object is turning at a constant rate, in degrees per second or revolutions per second, why do we have to consider that motion as acceleration?

In order to simplify things we can change our frame of reference when looking at angular motion. A change in the frame of reference is not limited to just the position we observe from but if we can define a set of mathematical functions to translate, or transform, from one frame of reference to another then we can simplify our understanding of the system by changing the frame of reference and use the transformation functions to move between frames of reference as we see fit - just as we did when considering loop speed when shooting and not shooting line.

When we consider angular motion and linear motion of an object we can draw parallels between the two types of motion by considering a rotation in terms of degrees of rotation as opposed to distance traveled. Thus the angular speed of an object is determined by how many degrees it rotates about a point in a given amount of time.

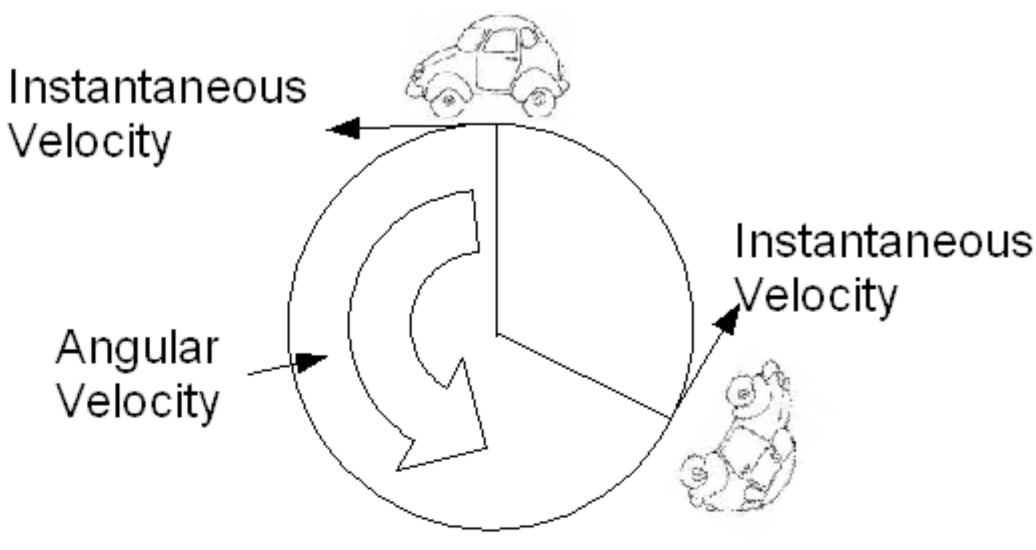




A car traveling around circular track of radius 1 mile in 15 minutes could be said to be traveling at approximately 25 miles per hour or 24 degrees per minute depending on our point of view. The second hand of a clock travels at 360 degrees per minute or one revolution per minute.

Angular velocity is determined by whether rotation is clockwise or counter clockwise. This gives us a magnitude (degrees or revolutions per second) and a direction which provides us an analogy between linear and angular speed and velocity. If we know how many degrees per second that an object is rotating and the radius of our rotation we can move between our linear and rotational frame of reference by using the formula: linear speed = (radius x 2 π x angular speed in degrees / 360) / time.

Converting velocity is a bit trickier. Any time an object is traveling around a circle its instantaneous velocity is tangential to its current location on the circle:



In order to determine the object's linear velocity we need to know its current location on the circle and then we can use trigonometry to determine its instantaneous direction of travel. The magnitude of the velocity is simply the speed as previously calculated.

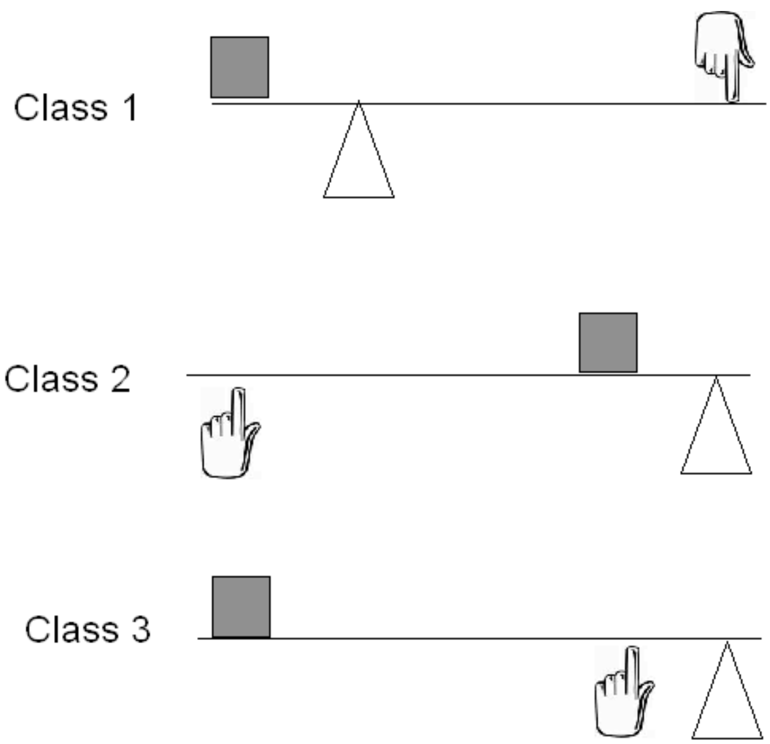
Angular acceleration (e.g., degrees per second per second) and linear acceleration also follow a similar relationship. If we denote linear acceleration by the symbol **a** and angular acceleration by the symbol **α**, then the relationship between the two is given by the formula: $a = \alpha \times \text{radius} \times 2\pi$. This gives us the conversion for magnitude of the linear acceleration. Again, in order to determine the instantaneous direction of the acceleration we need to know the current location of the object.

The final element we will consider at this time is angular force, or more correctly, torque. Torque can be thought of as a force that causes an object to turn or rotate. Torque can be measured in Newton meters, pound inches (or inch pounds), or pound feet (or foot pounds). Notice that units of measurement for torque consist of both a force and distance. In order to determine the amount of torque you multiply the amount of force being applied by the distance from the point of rotation. For example, if you apply 100 pounds of force to a nut using a 1 foot wrench you are applying 100 foot pounds of torque. If you apply the same amount of force but use a 2 foot wrench the amount of torque is 200 foot pounds.

Our discussion of torque leads us nicely into a discussion of levers.

Levers

Basically there are three types of levers called, conveniently, class 1, class 2 and class 3 as shown in the following figure.



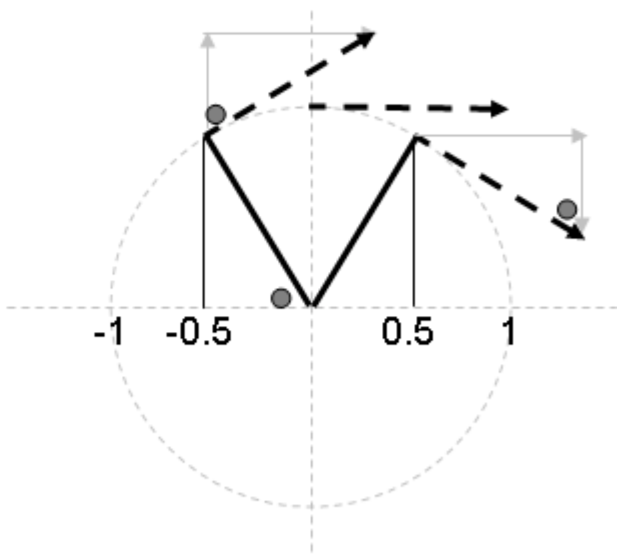
Each class of lever has the following elements in common – the lever is attached to a fulcrum (the triangle in our figure), a force is applied to a point on the lever (the hand), in order to move a load (the block). Since the lever is attached to the fulcrum it rotates about this point when we apply a force to the lever but as we saw in our discussion of angular movement a force that results in rotation is actually a torque. In addition, the amount of torque changes as we move the application point closer or farther to the fulcrum. So, if our point of application of force in a class 1 or 2 lever is 10 times farther from the fulcrum point than our load, we can move the load by applying only one tenth of the force generated by the load. If the load weighs 10 pounds we only need to exert 1 pound of force to move it.

The purpose of the class 1 and class 2 levers is obvious. We use them to make it easier to move heavy loads. But what purpose does the class 3 lever serve? We can see that to move a 10 pound load we would need to exert 100 pounds of force. Why would we want to do that? Again, notice that the movement of the class 3 lever is circular and that if we move our application point 1 inch then the load would move 10 inches. The class 3 lever allows us to move an object much faster than pushing on it directly. This principle applies to catapults, hockey sticks, and fly rods.

In the case of the fly rod consider a 9 foot rod held by an average person (distance from elbow to hand is one cubit or about 18"). If we move our hand 1 foot, by rotating at our elbow, the rod tip moves 7 feet. In theory, this lets us move the rod tip 7 times as fast as we are able to move our hand. As we saw in part 1 of this article this means that the kinetic energy transferred to the line is 49 times (remember that $E = ½ mv^2$) what we could have accomplished without the rod. The assumption here is that the mass of the load is much less than the amount of force that is being applied. If that condition is not met then the resistance of the load will prevent the multiplying effect from taking place.

Of course, obtaining 7 times the speed of our hand is a theoretical limit. There will be significant losses of acceleration of the line since the amount of resistance by the line is also multiplied by 7 and there are other losses due to air resistance (also multiplied) and other forms energy of loss. In addition, the path of the rod tip has a significant impact on the amount of energy that is imparted to the line.

In order to understand the impact of tip path and energy transfer let's consider two cases. In our first case the rod is rigid so the tip travels in a circular path as shown in the following figure:



$$x^2 + y^2 = 1$$
$$y = (1 - x^2)^{1/2}$$

Here we have the rod traveling in an arc of 60^0. The vector representing the force applied to the line at various points of the casting arc is shown as the dotted line arrows. The x and y component vectors at each of these points is represented by the gray colored arrows. As we can see that the x, or forward, component of force is at its maximum when the rod is in the vertical position and decreases as the rod moves away from this position. In order to calculate the amount of energy that is transmitted to the line from the rod we use the formula: W = E = F x d, i.e. Work = Energy = the amount of force that is applied and the distance over which the force is applied for. We can see from the figure that the x component of force is directly proportional to the value of y at each point. Since the forward component of the force is not constant over the path of the rod tip we must use integration to determine

$$\int \sqrt{1-x^2}\, dx =$$

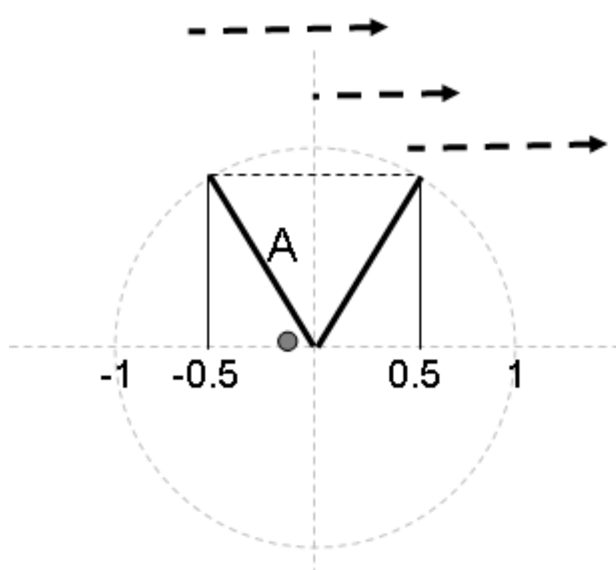
$$\frac{1}{2}\left(\sqrt{1-x^2}\; x + \sin^{-1}(x)\right)$$

how much force is applied as the rod tip travels from x = -0.5 to x = 0.5. We obtain the following results:

Substituting for x over the range -0.5 to 0.5 we find that the amount of energy that is transmitted to the line in the x direction is 0.774 of the total possible energy.

This tells us that for a circular path with a 60^0 arc the amount of energy transmitted to the line is roughly 77.4% of the total work exerted on the rod by our hand/arm/body. The remainder of the energy is lost in the y direction because the y component of force applied on the first half of the stroke is canceled out by the y component of force applied on the second half of the stroke. The actual value would be somewhat less as this calculation ignores loses due to other causes.

Now let's consider the same casting stroke but with a flexible rod that allows the tip to travel in a straight line path (SLP). This is shown in the following figure:



$$y^2 = 1 - 0.5^2$$

$$y^2 = 0.75$$

$$A = (x^2 + 0.75)^{1/2}$$

In this case all force is directed in the forward direction. But in order for the rod tip to travel in this SLP we can see that the effective length of the rod must be shortest when the rod is vertical. Remember from our discussion of torque that the effective amount of force applied to the line will also decrease as the effective length of the rod decreases. This means that the amount of force applied to the line is represented by the equation for A (the effective length of the rod). Again, applying an integration to the function we obtain the following result:

$$\int \sqrt{x^2 + 0.75}\, dx =$$

$$0.5 \sqrt{x^2 + 0.75}\; x + 0.375 \sinh^{-1}(1.1547\, x)$$

Substituting for x over the range -0.5 to 0.5 we find that the amount of energy that is transmitted to the line in the x direction is 0.912, i.e. 91.2%, of the total possible energy. We transmit 13.8% more energy to the line with a flexible rod than with a rigid rod if our casting arc is 60 degrees.

60^0 is a relatively narrow casting arc. The difference would be increased as the casting arc is increased. In addition the spring effect of the rod at the end of the stroke has not been factored in.

This concludes part two of this paper. Thank you.